$\mathbb{N}$	**Natural numbers**: $\mathbb{N} = \{1, 2, 3, \ldots\}$
$\mathbb{Z}$	**Integers:** $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Q}$	**Rational numbers**: $\mathbb{Q} = \{ a/b \mid a, b \in \mathbb{Z}, b \neq 0 \}$
$\mathbb{R}$	**Real numbers**: $\mathbb{R} = \{ \boldsymbol{x} \mid \boldsymbol{x}$ is a real number $\}$
$\varnothing$	**Empty Set:** $\varnothing = \{\}$
$\boldsymbol{U}$	**Universal set**
$\boldsymbol{x} \in \boldsymbol{X}$	**Set membership:** $\boldsymbol{x}$ is an element of $\boldsymbol{X}$
$\boldsymbol{x} \notin \boldsymbol{X}$	**Set non-membership:** $\boldsymbol{x}$ is not an element of $\boldsymbol{X}$
$\boldsymbol{X} = \boldsymbol{Y}$	**Set equality:** every element of $\boldsymbol{X}$ is an element of $\boldsymbol{Y}$ and vice versa $\boldsymbol{X} \subseteq \boldsymbol{Y}$ and $\boldsymbol{Y} \subseteq \boldsymbol{X}$
$\boldsymbol{X} \subseteq \boldsymbol{Y}$	**Set inclusion:** every element of $\boldsymbol{X}$ is an element of $\boldsymbol{Y}$; $\boldsymbol{X}$ is a **subset** of $\boldsymbol{Y}$ $\boldsymbol{X} \subseteq \boldsymbol{Y} \Leftrightarrow \forall \boldsymbol{x}$, if $\boldsymbol{x} \in \boldsymbol{X}$ then $\boldsymbol{x} \in \boldsymbol{Y}$
$\boldsymbol{X} \subset \boldsymbol{Y}$	**Proper subset**: $\boldsymbol{X} \subseteq \boldsymbol{Y}$, but $\boldsymbol{X} \neq \boldsymbol{Y}$
$\boldsymbol{X} \supseteq \boldsymbol{Y}$	
$\boldsymbol{X} \supset \boldsymbol{Y}$	
$\boldsymbol{X} \setminus \boldsymbol{Y}$	**Set theoretic complement**: $\{x \mid x \in X \wedge x \notin Y\}$
$\boldsymbol{X} \Leftrightarrow \boldsymbol{Y}$	***iff***
$\boldsymbol{X} \Rightarrow \boldsymbol{Y}$	***implies***
$\forall$	***for all***
$\exists$	***there exists***
$\ni$	***such that***
$\boldsymbol{X} \cap \boldsymbol{Y}$	**Intersection**: $\boldsymbol{X} \cap \boldsymbol{Y} = \{ \boldsymbol{x} \in \boldsymbol{U} \mid \boldsymbol{x} \in \boldsymbol{X} \wedge \boldsymbol{x} \in \boldsymbol{Y} \}$
$\boldsymbol{X} \cup \boldsymbol{Y}$	**Union:** $\boldsymbol{X} \cup \boldsymbol{Y} = \{ \boldsymbol{x} \in \boldsymbol{U} \mid \boldsymbol{x} \in \boldsymbol{X} \vee \boldsymbol{x} \in \boldsymbol{Y} \}$
$\{ \boldsymbol{a} \mid$ "statement about $\boldsymbol{a}$"$\}$	**Set comprehension:** the elements $\boldsymbol{a}$ which satisfy 'statement about $\boldsymbol{a}$'.
big-O	**Asymptotic upper bound** $f(n) = O(g(n)) \Leftrightarrow 0 \leq \lim_{x \to \infty} \left\| \frac{f(x)}{g(x)} \right\| < \infty$
little-o	**Asymptotically negligible** $f(n) = o(g(n)) \Leftrightarrow \lim_{x \to \infty} \left\| \frac{f(x)}{g(x)} \right\| = 0$
$\wedge$	**Logical AND**
$\vee$	**Logical OR**
$\oplus$	**Exclusive OR**
$\square$	QED (quad erat demonstratum)

$$\hat{\boldsymbol{a}} = a_x \hat{\boldsymbol{x}} + a_y \hat{\boldsymbol{y}} + a_z \hat{\boldsymbol{z}}$$

Vector

$$\hat{\boldsymbol{a}} \bullet \hat{\boldsymbol{b}} = a_x b_x + a_y b_y + a_z b_z$$

Dot Product

$$\hat{\boldsymbol{a}} \times \hat{\boldsymbol{b}} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Cross Product

$$\nabla f(x, y, z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

Gradient

$$\nabla \bullet \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Divergence

$$\nabla \times \mathbf{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Curl

$$\nabla^2 f(x, y, z) = \frac{\partial^2 f}{\partial x^2}\hat{x} + \frac{\partial^2 f}{\partial y^2}\hat{y} + \frac{\partial^2 f}{\partial z^2}\hat{z}$$

Laplacian

$\leq$

$\geq$

$\ll$

$\gg$

$\sim$

$\approx$

$\neq$

$\propto$

∞

Δ

∇

∇^2

$$\prod_{n=0}^{N} f(n)$$

$$\sum_{n=0}^{N} f(n)$$

$$\int_a^b f(x)dx$$

$$\frac{dy}{dx}$$

$$\frac{\partial y}{\partial x}$$

$$\lim_{x \to a}$$

$\bullet$

$\times$

Greek Alphabet		
Α	α	alpha
Β	β	beta
Γ	γ	gamma
Δ	δ	delta
Ε	ε	epsilon
Ζ	ζ	zeta
Η	η	eta
Θ	θ	theta
Ι	ι	iota
Κ	κ	kappa
Λ	λ	lambda
Μ	μ	mu
Ν	ν	nu
Ξ	ξ	xi
Ο	ο	omicron
Π	π	pi
Ρ	ρ	rho
Σ	σ or ς	sigma
Τ	τ	tau
Υ	υ	upsilon
Φ	φ or ϕ	phi
Χ	χ	chi
Ψ	ψ	psi
Ω	ω	omega

Matrices M X N (rows x columns)

One based indexing

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2n} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & \cdots & a_{mN} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & \cdots & a_{Mn} & \cdots & a_{MN} \end{bmatrix}$$

Zero based indexing

$$\begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0n} & \cdots & a_{0(N-1)} \\ a_{10} & a_{11} & \cdots & a_{1n} & \cdots & a_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_{m0} & a_{m1} & \cdots & a_{mn} & \cdots & a_{m(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{(M-1)0} & a_{(M-1)1} & \cdots & a_{(M-1)n} & \cdots & a_{(M-1)(N-1)} \end{bmatrix}$$

Set Theory

Group

A group is a set G together with a binary operation $\otimes$ which satify for all $x, y, z \in G$:

1. $(x \otimes y) \otimes z = x \otimes (y \otimes z)$ (associativity)
2. $\exists \quad I \in G \quad \ni \quad I \otimes x = x \otimes I = x$ (identity)
3. $\exists \quad x^{-1} \in G \quad \ni \quad x \otimes x^{-1} = x^{-1} \otimes x = I$ (inverse)
4. $x \otimes y \in G$ (closure – actually implied by "binary operation")

Ring

A ring is a set R together with two binary operators $\oplus$ and $\otimes$ which satify for all $x, y, z \in R$:

1. under $\oplus$ (Abelian group)
   a. $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ (associativity)
   b. $\exists \quad 0 \in R \quad \ni \quad 0 \oplus x = x \oplus 0 = x$ (identity)
   c. $\exists \quad -x \in R \quad \ni \quad x \oplus (-x) = (-x) \oplus x = 0$ (inverse)
   d. $x \oplus y = y \oplus x$ (commutativity - Abelian)
2. under $\otimes$ (semigroup)
   a. $(x \otimes y) \otimes z = x \otimes (y \otimes z)$ (associativity)
3. $\otimes$ distributes over $\oplus$ (right and left)

   $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$ and $(y \oplus z) \otimes x = (y \otimes x) + (z \otimes y)$

Field

A field is a set F together with two binary operators $\oplus$ and $\otimes$ which satify for all $x, y, z \in R$:

1. under $\oplus$ (Abelian group)
   a. $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ (associativity)
   b. $\exists \quad 0 \in R \quad \ni \quad 0 \oplus x = x \oplus 0 = x$ (identity)
   c. $\exists \quad -x \in R \quad \ni \quad x \oplus (-x) = (-x) \oplus x = 0$ (inverse)
   c. $x \oplus y = y \oplus x$ (commutativity - Abelian)
2. under $\otimes$ (Abelian group except for zero restriction on inverse)
   a. $(x \otimes y) \otimes z = x \otimes (y \otimes z)$ (associativity)
   b. $\exists \quad 1 \in R \quad \ni \quad 1 \otimes x = x \otimes 1 = x$ (identity)
   c. $\forall x \neq 0 \quad \exists \quad x^{-1} \in R \quad \ni \quad x \otimes (x^{-1}) = (x^{-1}) \otimes x = 1$ (inverse)
   d. $x \otimes y = y \otimes x$ (commutativity)
3. $\otimes$ distributes over $\oplus$ (right and left)

   $x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$ and $(y \oplus z) \otimes x = (y \otimes x) + (z \otimes y)$

Abelian Group

A group for which the elements commute.

Subgroup

A subset of a group which is closed under the group operation.

Semigroup

A semigroup is a set together with a binary operator in which the operation is associative.

(Note – need not have an identiy element nor inverse).

Binary operation

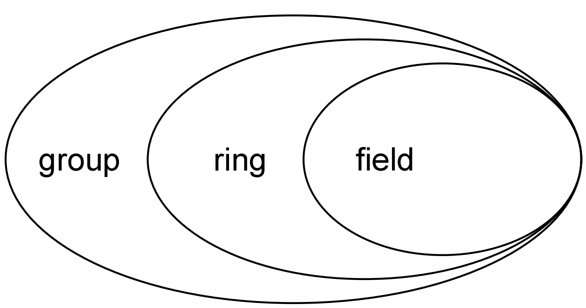
A binary operation on a nonempty set A is a map $f : A \times A \to A$ such that

1. f is defined for every pair of elements in A
2. f uniquely associates each pair of elements in A to some element of A

Closure

A mathematical structure A is said to be closed under an operation + if, whenever a and b are both elements of A, then so is $a+b$.

Notes on set theory



Topology

Complete Metric Space

A **metric space** in which every **Cauchy sequence converges** (to an element in the space).

Examples include the real numbers with the usual metric, the complex numbers, finite-dimensional real and complex vector spaces, and the space of square-integrable functions on the unit inverval $L^2([0,1])$.

Metric

A binary function $g(x, y)$ for a given set satisfying the following conditions:

1. $g(x, y) \geq 0$ (nonnegative)

   1a. $g(x, y) = 0 \Leftrightarrow x = y$.

2. $g(x, y) = g(y, x)$ (symmetry)

3. $g(x, y) + g(y, z) \geq g(x, z)$ (triangle inequality)

A set possessing a metric is called a **metric space**.

Cauchy Sequence

A sequence $a_1, a_2, \ldots$ such that the metric $d(a_m, a_n)$ satisfies $\lim_{\min(m,n)\to\infty} d(a_m, a_n) = 0$.

Cauchy sequences in the rationals do not necessarily converge, but they do converge in the reals.

Convergent Sequence

A sequence S_n converges to the limit $\lim_{n\to\infty} S_n = S$ if, for any $\varepsilon > 0$, there exists an N such that $\left|S_n - S\right| < \varepsilon$ for all $n > N$. If S_n does not converge, it is said to diverge.

Every bounded monotonic sequence converges. Every unbounded sequence diverges.

Inner Product

An inner product is a generalization of the dot product. In a vector space, it is a way to multiply vectors together, with the result of this multiplication being a scalar.

More precisely, for a real vector space, and inner produce $\langle \cdot, \cdot \rangle$ satisfies the following four properties. Let u, v and w be vectors and α be a scalar, then:

1. $\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$
2. $\langle \alpha v, w \rangle = \alpha \langle v, w \rangle$
3. $\langle v, w \rangle = \langle w, v \rangle$
4. $\langle v, v \rangle \geq 0$ and equal iff $v = 0$

A vector space together with an inner product on it is called an **inner product space**. This definition also applies to an **abstract vector space** over any field.

Examples of inner product spaces include:

1. The real numbers $\mathbb{R}$, where the inner product is given by $\langle x, y \rangle = xy$.
2. The Euclidean space $\mathbb{R}^n$, where the inner product is given by the dot product $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \sum_{i=1}^{n} x_i y_i$.
3. The vector space of real functions whose domain is a closed interval $[a, b]$ with the inner product

$$\langle f, g \rangle = \int_a^b f\, g\, dx$$

When given a complex vector space, the third property above is usally replaced by

$$\langle v, w \rangle = \overline{\langle w, v \rangle}$$

where $\bar{z}$ refers to complex conjugation. With this property, the inner product is called a **Hermitian inner product** and a complex vector space with a Hermitian inner product is called a **Hermitian inner product space**.

Every inner product space is a metric space. The metric is given by $g(v, w) = \langle v-w, v-w \rangle$.

If this process results in a **complete metric space**, it is called a **Hilbert space**.

Complete Metric

A complete metric is a metric in which every Cauchy sequence is convergent. A topological space with a complete metric is called a complete metric space.

Inner Product Space

A **vector space** together with an **inner product** on it. If the inner product defines a **complete metric**, then the inner product space is called a **Hilbert space**.

Vector Space

A vector space V is a set that is closed under finite vector addition and scalar multiplication and the following conditions hold for all elements $\boldsymbol{x}, \boldsymbol{y}, z \in V$ and any scalars $r, s \in F$:

1. Commutativity: $\boldsymbol{x} + \boldsymbol{y} = \boldsymbol{y} + \boldsymbol{x}$
2. Associativity of vector addition: $(\boldsymbol{x} + \boldsymbol{y}) + z = \boldsymbol{x} + (\boldsymbol{y} + z)$
3. Additive identity $\forall \boldsymbol{x}: \quad 0 + \boldsymbol{x} = \boldsymbol{x} + \boldsymbol{0} = \boldsymbol{x}$
4. Existance of additive inverse: $\forall \boldsymbol{x} \quad \exists \boldsymbol{-x} \quad \ni \quad \boldsymbol{x} + (\boldsymbol{-x}) = \boldsymbol{0}$
5. Associativity of scalar multiplication: $r(s\boldsymbol{x}) = (rx)\boldsymbol{x}$
6. Distributivity of scalar sums: $(r + s)\boldsymbol{x} = r\boldsymbol{x} + r\boldsymbol{x}$
7. Distributivity of vector sums: $r(\boldsymbol{x} + \boldsymbol{y}) = r\boldsymbol{x} + r\boldsymbol{y}$
8. Scalar multiplication identity: $1\boldsymbol{x} = \boldsymbol{x}$

The basic example is n-dimensional Euclidean space $\mathbb{R}^n$, where every element is represented by a list of n real numbers, scalars are real numbers, addition is componentwise, and scalar multiplication is multiplication on each term separately.

For a general vector space, the scalars are members of a field *F*, in which case *V*, is called a vector space over *F*.

Euclidean n-space $\mathbb{R}^n$ is called a real vector space, and $\mathbb{C}^n$ is called a complex vector space.

Norm

The norm of a mathematical object is a quantity that in some (possibly abstract) sense describes the length, size, or extent of the object.

Given a vector space V over *K*, a norm on a vector space *V* is a function $\|\bullet\| : V \to \mathbb{R}; x \to \|x\|$ with the following properties:

1. $\|\boldsymbol{v}\| \geq 0$, with equality iff $\boldsymbol{v} = 0$ (non-negative)
2. $\|a\boldsymbol{v}\| = |a|\|\boldsymbol{v}\|$ (multiplication by scalar)
3. $\|\boldsymbol{v} + \boldsymbol{u}\| \leq \|\boldsymbol{v}\| + \|\boldsymbol{u}\|$ (triangle inequality)

Most commonly "norm" refers to the vector norm

$$|\boldsymbol{x}| = \sqrt{x_1^{\,2} + x_2^{\,2} + \ldots + x_n^{\,2}}$$

Hilbert Space

A Hilbert space is an inner product space that is complete with respect to the norm defined by the inner product.

A Hilbert space is always a Banach space, but the converse need not hold.

Banach Space

A Banach space is a complete vector space B with a norm $\|\cdot\|$. Two norms $\|\cdot\|_{(1)}$ and $\|\cdot\|_{(2)}$ are called equivalent if they give the same topology, which is equivalent to the existence of constants c and C such that

$$\|v\|_{(1)} \le c\|v\|_{(2)} \text{ and } \|v\|_{(2)} \le C\|v\|_{(1)}$$

hold for all v.

In the finite-dimensional case, all norms are equivalent. An infinite-dimensional space can have many different norms.

A basic example is n-dimensional Euclidean space with the Euclidean norm. Usually the notion of Banach space is only used in the infinite dimensional setting, typically as a vector space of functions. For example, the set of continuous functions on the real line with the norm of a function f given by

$$\|f\| = \sup_{x\in\mathbb{R}} |f(x)|$$

is a Banach space.

On the other hand, the set of contunuous functions on the unit interval $[0,1]$ with the norm of a function f given by

$$\|f\| = \int_0^1 |f(x)|\,dx$$

is not a Banach space, because it is not complete. For instance, the Cauchy sequence of functions

$$f_n = \begin{cases} 1 & \text{for } x \le \frac{1}{2} \\ \frac{1}{2}n + 1 - nx & \text{for } x \le \frac{1}{2} + \frac{1}{n} \\ 0 & \text{for } x > \frac{1}{2} + \frac{1}{n} \end{cases}$$

does not converge to a continuous function.

Hilbert spaces with their norm given by the inner product are examples of Banach spaces. While a Hilbert space is always a Banach space, the converse need not hold. Therefore, it is possible for a Banach space not to have a norm given by an inner product. For instance, the supremum norm cannot be given by an inner product.

Vector norms defined for a complex vector $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T$ ($|x_k|$ denotes the complex modulus)

l^1-norm

$$|\boldsymbol{x}|_1 = \sqrt{\sum_{k=1}^{n} |x_k|}$$

l^2-norm

$$|\boldsymbol{x}|_2 = \sqrt{\sum_{k=1}^{n} |x_k|^2}$$

l^p-norm

$$|\boldsymbol{x}|_p = \sqrt{\sum_{k=1}^{n} |x_k|^p}$$

l^∞-norm

$$|\boldsymbol{x}|_\infty = \max_i |x_i|$$

Function norms

L^2-norm

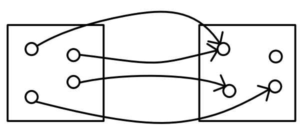
Applied to a function y(x): $|y|^2 \equiv \langle y \mid y \rangle \equiv \int |y(x)|^2 \, dx$, where $\langle f \mid g \rangle$ denotes the angle bracket.

Mappings

Morphism/map

A way of associating unique objects to every point in a given set. A morphism $f : A \rightarrow B$ from A to B is a function f such that for every $a \in A$, there is a unique object $f(a) \in B$.

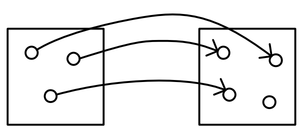
$$f : A \rightarrow B \quad \Rightarrow \quad \forall\, a \in A \quad \exists \quad \text{a unique} \quad f(a) \in B \qquad (\textbf{morphism/map})$$



Injection/One-to-one

A mapping $f(x)$ such that whenever $f(x) = f(y)$, it must be the case that $x = y$. In other words a mapping is an injection if it maps distinct objects to distinct objects.

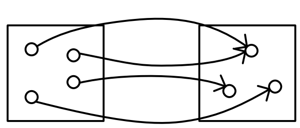
$$f : A \rightarrow B \quad | \quad f(x) = f(y) \quad \Rightarrow \quad x = y \qquad (\textbf{injection/one-to-one})$$



Surjection/onto

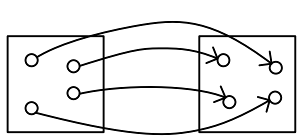
A mapping $f(x)$ such that for any $b \in B$, there exists an $a \in A$ for which $b = f(a)$.

$$f : A \rightarrow B \quad | \quad \forall b \in B \quad \exists\, a \in A \quad \ni \quad b = f(a) \qquad (\textbf{surjection/onto})$$



Bijection /Isomorphism

A mapping $f(x)$ which is **one-to-one** and **onto**.



Note – bijections have inverses.

Homeomorphic

Continuous, one-to-one, onto, and having a continuous inverse.

Discrete Fourier Transform

$$f_k = \frac{1}{N}\sum_{n=0}^{N-1} F_n e^{2\pi ikn/N}$$

$$F_n = \sum_{k=0}^{N-1} f_k e^{-2\pi ink/N}$$

Fourier Series

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi nx}{L}) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi nx}{L})$$

$$a_0 = \frac{2}{L}\int_{-L/2}^{L/2} f(x)dx$$

$$a_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\cos(\frac{2\pi nx}{L})dx$$

$$b_n = \frac{2}{L}\int_{-L/2}^{L/2} f(x)\sin(\frac{2n\pi x}{L})dx$$

or

$$f(x) = \sum_{n=-\infty}^{\infty} A_n e^{i(2\pi nx/L)}$$

$$A_n = \frac{1}{L}\int_{-L/2}^{L/2} f(x)e^{-i(2\pi nx/L)}dx$$

Fourier Transform

$$h(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{i\omega t}d\omega$$

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-i\omega t}dt$$

Laplace Transform

$$f(t) = \tfrac{1}{i2\pi}\int_{c-j\infty}^{c+j\infty} e^{st}H(s)ds$$

$$H(s) = \int_{0}^{\infty} f(t)e^{-st}dt$$

z-Transform

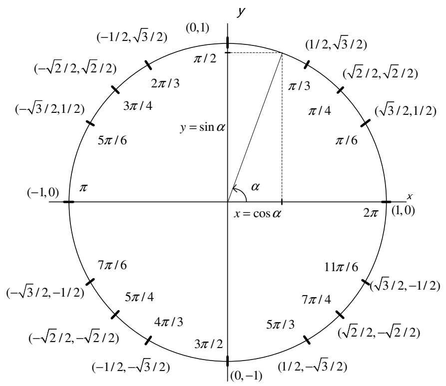
$$x[n] = \oint_C X(z)z^{n-1}dz$$

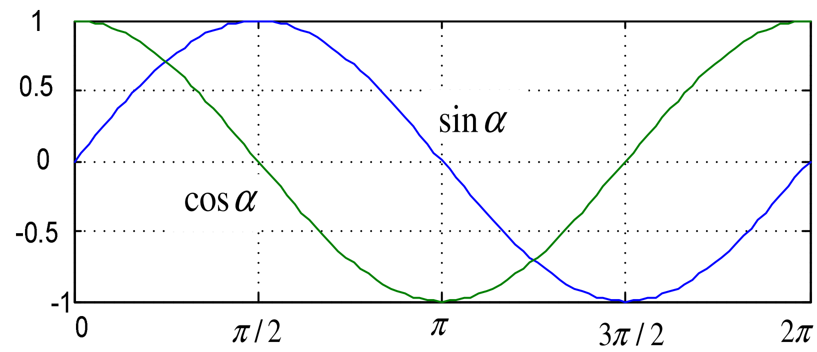
$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

Trigonometry

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} \qquad \cos\alpha = \frac{\text{adj}}{\text{hyp}} \qquad \tan\alpha = \frac{\text{adj}}{\text{opp}} = \frac{\sin\alpha}{\cos\alpha}$$

$$\csc\alpha = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\alpha} \qquad \sec\alpha = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\alpha} \qquad \cot\alpha = \frac{\text{opp}}{\text{adj}} = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$$





Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

Law of sines

$$\frac{\sin a}{A} = \frac{\sin b}{B} = \frac{\sin c}{C}$$

Taylor's Series Expansion

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

Identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

$$\tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$

$$\cot\alpha + \cot\beta = \frac{\sin(\alpha+\beta)}{\sin\alpha\sin\beta}$$

$$\cot\alpha - \cot\beta = \frac{-\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$$

Euler's formula

$$e^{i\alpha} = \cos\alpha + i\sin\alpha$$

Derivative

$$f'(x) = \frac{df(x)}{dx} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

Integral

Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a) \quad \text{where} \quad F'(x) = f(x)$$

Integration by Parts

$$\int_a^b f(x)g'(x)dx = f(x)g(x)\Big|_a^b - \int_a^b g(x)f'(x)dx \qquad \textbf{or} \quad \int u\,dv = uv - \int v\,du$$

Substitution Rule

$$\int_{\phi(a)}^{\phi(b)} f(x)dx = \int_a^b f(\phi(t))\phi'(t)dt$$

Common Derivatives

$$\frac{d}{dx}cx = c$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}[f+g] = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx}[fg] = f\frac{dg}{dx} + \frac{df}{dx}g$$

$$\frac{d}{dx}\frac{f}{g} = \frac{\frac{df}{dx}g - f\frac{dg}{dx}}{g^2}$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Common Integrals

$$\int dx = x + C$$

$$\int cf(x)dx = c\int f(x)dx$$

$$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int \frac{1}{x}dx = \ln|x| + C$$

$$\int \frac{1}{1+x^2}dx = \arctan x + C$$

$$\int \ln x dx = x\ln x - x + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int \sin x\,dx = -\cos x + C$$

$$\int \cos x\,dx = \sin x + C$$